

# PROBABILITY

Thabisa Tikolo

STATISTICS SOUTH AFRICA

*Probability is a topic that some educators tend to struggle with and thus avoid teaching it to learners. This is an indication that teachers are not yet completely comfortable with the topic. This workshop is an attempt to develop teachers' content knowledge and understanding of Probability in grades 7 -9 as it forms the basis for all the Probability concepts that they will be dealing with in the FET phase- both in Mathematics and Maths Lit.*

In this workshop relevant Data from both Census@Schools 2009 and Census 2011 will be used

## PROBABILITY IN THE GET PHASE

### Relative frequency and probability

During this workshop teachers will: List all the possible outcomes of simple experiments, determine the relative frequency of actual outcomes for a series of trials, Determine the probability of outcomes of simple situations using the definition for probability, predict the frequency of possible outcomes based on their probability, Compare relative frequency and probability and explain possible differences, determine probabilities for compound events using two-way tables and tree diagrams  
A complete memo with all the answers to the examples from the hand out will be provided to every participant at the end of the workshop.

### Proposed time allocation for workshop activities:

1. List all the possible outcomes of simple experiments	20 minutes
2. Determine the relative frequency of actual outcomes for a series of trials	20 minutes
3. Determine the probability of outcomes of simple situations using the definition for probability	20 minutes
4. Predict the frequency of possible outcomes based on their probability.	20 minutes
5. Compare relative frequency and probability and explain possible differences.	20 minutes
6. Determine probabilities for compound events using two-way tables and tree diagrams	20 minutes

## ACTIVITY 1 – RECAP

Probability– is the part of mathematics that studies chance or likelihood. For example, when you see on the weather report that there is a 60% chance of rain, they are talking about a probability.

### 1.1 Explanation of terms:

**Experiment** – one or more activities to see what happens e.g. tossing a coin, throwing a dice is an activity or experiment.

**Sample Space** – made up of all possible outcomes of an experiment. E.g. Sample space of tossing a coin =  $S = \{H, T\}$

**Event** - An outcome or combination of outcomes that we are interested in e.g. getting tails when tossing a coin is an event.

**Favourable outcomes** – outcome or combination of outcomes of an event that we are interested in

### 1.2 Probability Scales

Rather than using words to describe the chance of an event happening, you can give probability as a number **between 0 and 1**.

- This number can be written as a fraction, percentage or decimal
- If it is **impossible** for an event to happen, the probability is **0**.
- If an event is **certain** to happen, the probability is **1**.
- All other probabilities are **greater than 0, but less than 1**.

*Examples:*

Some events **always happen**. We say that they are certain to happen and give them a probability of **1**.

- It is **certain** that the day after Monday is Tuesday
- The probability that the day after Monday is Tuesday is **1**.
- Some events **never happen**. We say that they are **impossible** and give them a probability of **0**.
- If you throw an ordinary dice, it is impossible to get a 7?
- The probability of getting a 7 when you throw an ordinary dice is 0.
- Some events are **not certain**, but are **not impossible either**. They may or may not happen. These probabilities lie **between 0 and 1**.
- If you toss a fair coin it may land on heads or it may not. The chances are **equally likely**.

### 1.3 ACTIVITY: LISTING OUTCOMES OF AN EXPERIMENT:

- If we do a probability experiment, for example throw a die, toss a coin, spin a spinner, we can list all the possible outcomes.
- An event is a particular outcome or group of outcomes.
- Favorable outcomes are the outcomes which give the event you are interested in.

- If the die is fair, then we can say that the outcomes are equally likely. In other words all numbers have the same chance of being thrown. One number doesn't have a greater chance than the others of being thrown.
- If a die is NOT fair, we say that it is **biased**.

### Exercise 1 A

Suppose you throw a fair die.

- 1) List all the possible outcomes
- 2) List a favorable outcome for the event:
  - a) Getting a 4
  - b) Getting an even number



### Exercise 1.B

An eight-sided die (like the one alongside) is thrown

- a) List all the possible outcomes.
- b) List all the favourable outcomes for the following



events:

**Event A:** Getting a 2

**Event B:** Getting an odd number

**Event C:** Getting a number bigger than 4.

### Exercise 1.C

A coin is randomly taken from this purse.

Inside the purse are a R5 coin, a R2 coin, two R1 coins, and a 20c coin.



- a) List all the possible outcomes.
- b) List all the favourable outcomes for the following events:

**Event D:** Getting a R1 coin

**Event E:** Getting a 'bronze' coin

**Event F:** Getting a coin worth more than R4.

### ACTIVITY2. FINDING THE RELATIVE FREQUENCY:

We find the relative frequency of an experiment by performing the experiment or by collecting information from past records

Relative frequency can be calculated using the following formulae:

$$\text{Relative frequency} = \frac{\text{no. of times an outcome occurs in an experiment}}{\text{total number of trials in the experiment}}$$

OR

$$\text{Relative Frequency} = \frac{\text{no. of times an outcome occurs in a survey}}{\text{total number of observations in a survey}}$$

### Example:

In an experiment a drawing pin is dropped 100 times.

It lands with its **point up** 37 times.

What is the relative frequency of the drawing pin landing point up?

*Solution:*

$$\text{Relative frequency} = \frac{\text{no. of times the drawing pin lands point up}}{\text{total number of times the drawing pin is dropped}} = \frac{37}{100} = 0,37 \text{ or } 37\%$$

### Exercise 2.1

1) In an experiment a gardener planted 40 daffodil bulbs.



36 of these daffodil bulbs produced flowers.

Use these results to find the relative frequency that a daffodil bulb will produce a flower. Use the formula:

$$\text{Relative frequency} = \frac{\text{no. of bulbs producing flowers}}{\text{total number of bulbs planted}}$$

### Exercise 2.2



Thandi keeps a record of her chess games with Helen. Out of the first 30 games, Helen wins 21 games.

- a) Use the results to work out
  - (i) The relative frequency that Helen wins
  - (ii) Use the two relative frequencies to predict whether Thandi will win her next game of chess with Helen.

### ACTIVITY3. CALCULATING PROBABILITY:

In the previous section we looked at how you could estimate probabilities by doing experiments or making observations.

When we have a situations where each outcome is **equally likely** to occur, we can calculate the **probability** of a particular outcome occurring.

**Probability** can be calculated using the following formula:

$$\text{Probability} = \frac{\text{no. of favourable outcomes in an event}}{\text{total number of possible outcomes}}$$

#### Example:

An ordinary die is rolled. What is the probability of getting?

- a) A 6?
- b) An odd number?
- c) A 2 or 3?



**Solution:**

The possible outcomes on an ordinary die are: 1; 2; 3; 4; 5; and 6.

The total number of possible outcomes is 6.

a) The favourable outcome is 6.

The number of favourable outcomes is 1

$$P(6) = \frac{1}{6} \approx 0,17 \approx 17,7\%$$

b) The favourable outcomes are 1; 3 and 5.

The number of favourable outcomes is 3

$$P(\text{odd}) = \frac{3}{6} = \frac{1}{2} \approx 0,5 \approx 50\%$$

c) The favourable outcomes are 2 and 3.

The number of favourable outcomes is 2

$$P(2 \text{ or } 3) = \frac{2}{6} = \frac{1}{3} \approx 0,33 \approx 33\%$$

**Exercise: 3.1**

1) A bag contains a red counter, a blue counter and a green counter.

A counter is taken from the bag at random.

a) What are the possible outcomes?

b) What is the probability of taking?

(i) A red counter?






































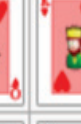














(ii) A red or green counter?

(iii) A counter that is not blue?

(iv) A yellow counter?

2) A card is randomly taken from a full pack of 52 playing cards with no jokers

Example set of 52 poker playing cards

Suit	Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Clubs													
Diamonds													
Hearts													
Spades													

- Thirteen of the cards are marked with a black spade (♠)
- Thirteen of the cards are marked with a red heart (♥)
- Thirteen of the cards are marked with a red diamond (♦)
- Thirteen of the cards are marked with a black club (♣)
- Each set of thirteen cards is called a **suit**
- The 1 is generally marked A and is called the Ace
- The picture cards are **Jack, Queen** and **King**.

What is the probability that the card that is taken is:

- a) A red card?
- b) A heart?
- c) The Ace of hearts?

**NOTE:**

When a question asks you to estimate the probability, it is actually asking you to calculate the relative frequency

**The larger the number of trials or observations, the closer the relative frequency is to the probability.**

**ACTIVITY4. PREDICTING FREQUENCIES BASED ON PROBABILITY**

We can use the probability of an outcome occurring to predict or estimate the frequency of that outcome occurring in an experiment.

We can use the formula to estimate the frequency of an outcome occurring:

***Predicted frequency of an outcome = Probability of that outcome occurring × Number of repeats of the experiment***

**Example**



A fair coin is tossed 10 times. How many heads could you expect to get?

*Solution:*

**The probability of getting heads =  $\frac{1}{2}$**

**Predicted frequency of heads = probability of getting a head × number of times the coin is tossed**

$$\frac{1}{2} \times 10 = 5$$

**Exercise 4.1**

- 1) There are 50 cars in a school car park. Five of the cars are black.
  - a) What is the probability that the first car to leave the car park will be black?
  - b) If the probability that a red car is the first to leave is 0, 2, how many red cars are there in the car park?

### Exercise 4.2

500 tickets are sold for a prize draw.

- Sam buys one ticket. What is the probability that he wins the first prize?
- The probability of Cynthia winning first prize is  $\frac{1}{20}$ . How many tickets did she buy?

### ACTIVITY 5. PROBABILITIES OF COMPOUND EVENTS OCCURRING:

So far, we have been looking at probabilities of single events occurring.

In some problems, we will have to find the probability of compound events occurring. Events are called compound events when two (or more) activities take place.

Examples of compound events:

- We spin a spinner and then select a card.
- We throw two dice together
- We toss a coin twice.

In situations like these, we can find all the possible outcomes using:

- A list
- A two-way table
- A tree diagram.

#### Example

A fair coin is tossed twice.

- Identify all the possible outcomes
- Find the probability of getting two heads

#### Solution

(i) List the outcomes systematically.

1 <sup>st</sup> throw	2 <sup>nd</sup> throw
Head (H)	Head (H)
Head (H)	Tail (T)
Tail (T)	Head (H)
Tail (T)	Tail (T)

(ii) Use a two-way table.

		1 <sup>st</sup> throw	
		H	T
2 <sup>nd</sup> throw	H	HH	TH
	T	HT	TT

(iii) Use a tree diagram

1 <sup>st</sup> throw	2 <sup>nd</sup> throw	Outcome
H	H	→ HH
	T	→ HT
T	H	→ TH
	T	→ TT

- All three of these methods show that there are four possible outcomes, namely: HH, HT, TH and TT.

- There is only one favourable outcome in the event of getting two heads, so:

$$P(\mathbf{H; H}) = \frac{1}{4}$$

### Exercise 5.1

1) A fair coin is tossed and an ordinary die is rolled

a) Copy and complete the two-way table to list all the possible outcomes

		Die					
		1	2	3	4	5	6
Coin	H						
	T						

b) Use the table to calculate the probability of getting:

(i) A head and 5?

(ii) A tail and 6?

(iii) A tail and an even number?

(iv) A tail and an odd number?

(v) A head and a number greater than 3?

(vi) An odd number

### Exercise .5.2

Mbali has 3 pairs of jeans: one blue, one black and one red. She also has 4 shirts: one blue, one white, one black and one striped.

a) Draw a tree diagram to show how many outfits she can 'mix and match' with these clothes

b) If she chose a shirt and a pair of jeans with her eyes shut what is the probability that she would choose the black jeans and the black shirt?

### REFERENCES:

Census@School (2013). Data Handling and Probability (Grade 10, 11 and 12). Statistics South Africa.

Pike M. et al. (2011). Classroom Mathematics Grade 10. Heinemann.

Graham D. and Graham C. (1996). Mainstream Mathematics for GCSE. Macmillan Press LTD.

Speed B. (1997). Higher Mathematics for GCSE. Collins Educational